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A simple model of Aharonov–Berry’s superoscillations

Wang Qiao

Department of Mathematics, Wuhan University, Wuhan, 430072, People’s Republic of China
and

Information Science Center, Peking University, Beijing, 100871, People’s Republic of China

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Abstract. A simple method of constructing superoscillatory functions is given. It is based on a zero-shifting principle of functions in Paley–Wiener space.

There exist functions that oscillate like $\cos x$ but they can do so arbitrarily faster than $\cos x$ in an interval with arbitrary length. This strange phenomenon, called superoscillation, is firstly described by Aharonov (cf [1]). Using the asymptotic of integrals, Berry gave a clever way to construct a large class of superoscillation functions ([1]). This fact has many interesting applications in quantum mechanics such as [2] and [3]. In this paper we will give a zero-shifting principle which implies that we can get arbitrary superoscillation from any band-limited function. Any physical signal has finite energy, so we consider the function $f(x) \in L^2(\mathbb{R})$ with $\widehat{f}(\xi) = 0$ for $|\xi| \geq \pi$, where \widehat{f} means Fourier–Laplace transform. These functions form a Paley–Wiener space W_π . The following theorem is classical (cf [4]).

Theorem (Paley–Wiener). Suppose $\xi \in \mathbb{R}$. If $\widehat{g}(\xi) \in L^2([-\pi, \pi])$ then $\widehat{g}(\xi) = 0$ for $|\xi| \geq \pi$ if and only if there is a $f(x) \in L^2(\mathbb{R})$, $f(z)$ is entire and $f(z) = O(e^{\pi|z|})$ such that $\widehat{f}(\xi) = \widehat{g}(\xi)$.

Now we can give

Theorem (Zero-shifting principle). If $f(x) \in W_\pi$ is continuous with c zero, then for any real number s , $g(x) = \frac{x-s}{x-c} f(x)$ is also in W_π .

Proof. $g(z) = \frac{z-s}{z-c} f(z)$ is of $O(e^{\pi|z|})$ and $g(x) \in L^2(\mathbb{R})$ since $g(z) \sim f(z)$ for $|z| \rightarrow \infty$.

Using this theorem we can get arbitrary superoscillation functions from arbitrary given band-limited functions very easily. For example, consider

$$f(x) = \frac{\sin \pi x}{\pi x} = \prod_{n=1}^{\infty} \left(1 - \frac{x^2}{n^2}\right)$$

which oscillates like $\cos \pi x$. Slightly modifying it, we get

$$g(x) = \prod_{n=1}^N \left(1 - \frac{k^2 x^2}{n^2}\right) \prod_{n=N+1}^{\infty} \left(1 - \frac{x^2}{n^2}\right) \quad k > 1$$

which oscillates like $\cos kx$ during $[-N/k, N/k]$ but globally like $\cos x$.

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